# REPORT No. 299

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# INVESTIGATION OF DAMPING LIQUIDS
# FOR AIRCRAFT INSTRUMENTS

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Bureau of Standards

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## SUMMARY

*This report covers the results of an investigation carried on at the Bureau of Standards with the financial assistance of the National Advisory Committee for Aeronautics.*

*The choice of a damping liquid for aircraft instruments is difficult owing to the range of temperature at which aircraft operate. Temperature changes affect the viscosity tremendously. The investigation was undertaken with the object of finding liquids of various viscosities otherwise suitable which had a minimum change in viscosity with temperature. The new data relate largely to solutions.*

*The effect of temperature on the kinematic viscosity of the following liquids and solutions was determined in the temperature interval $-18°$ to $+30°$ C.*

*(1) Solutions of animal and vegetable oils in xylene. These were poppy seed oil, two samples of neat's-foot oil, castor oil and linseed oil.*

*(2) Solutions of mineral oils in xylene. These were Squibb's petrolatum of naphthene base and transformer oil.*

*(3) Glycerine solutions in ethyl alcohol and in mixture of 50–50 ethyl alcohol and water.*

*(4) Mixtures of normal butyl alcohol with methyl alcohol.*

*(5) Individual liquids; kerosene, mineral spirits, xylene, and recoil oil.*

*The apparatus consisted of four capillary-tube viscometers, which were immersed in a liquid bath in order to secure temperature control. The method of calibration and the related experimental data are presented in detail.*

*The viscosity data for the liquids are given in curves in which $log_{10} \dfrac{t_\theta}{t_{30}}$ is plotted against temperature, where $t_{30}$ and $t_\theta$ are respectively the times of discharge through the viscometer at $30°$ C. and $\theta°$ C. Except for a correction which is usually small, the following relation holds:*

$$log_{10}\frac{t_\theta}{t_{30}} = log_{10}\frac{\nu_\theta}{\nu_{30}}$$

*in which $\nu_{30}$ and $\nu_\theta$ are respectively the kinematic viscosities at $30°$ C. and $\theta°$ C. The density at $30°$ C., the coefficient of cubical thermal expansion for each solution, and $\nu_{30}$ are given, together with other data, so that the absolute viscosity may be computed. The accuracy is within 1 per cent.*

## INTRODUCTION

This investigation was undertaken in order to obtain data on the viscosity and the change in viscosity with temperature of such liquids and solutions as may be suitable for damping purposes in aircraft instruments. Damping liquids in many cases must also have other characteristics, such as invariability with time, low volatility, and constancy of index of refraction and transparency for varying temperatures. These latter characteristics are not dealt with in this paper except incidentally.

Most of the liquids which were chosen for the tests are solutions of mineral, vegetable, and animal oil in xylene. The selection of xylene as a solvent for the heavier oils finds its reason in the fact that it has a low thermal coefficient of viscosity for the thermal range considered in

aircraft instruments, which is the range $+30°$ C. to $-30°$ C. The range of temperatures in the tests was somewhat smaller than this and was from $+30°$ C. to $-18°$ C.

An accuracy of 1 per cent was aimed at in the determination of viscosity at the temperature of $30°$ centigrade. Two considerations have discouraged possible effort for greater precision: First, it is a known fact that most oily solutions age with time, and therefore the viscosity of such solutions need not be accurately determined except for the purpose of showing that the viscosity changes (Reference 1); secondly, more precise determinations of viscosity for the particular range adopted can not be realized with a reasonable amount of attention and effort.

It is desired to acknowledge the valuable assistance of Dr. W. G. Brombacher and the substantial help of Mr. E. R. Melton in obtaining the experimental data.

## APPARATUS AND EXPERIMENTAL PROCEDURE

### VISCOMETERS

Consider the particular Ostwald-type viscometer which is shown in Figure 1. The discharging and receiving bulbs $A$ and $B$ are cylinders of equal radius and volume. The timing of the flow is made for the total discharge of liquid in the discharging bulb $A$. When this total discharge has taken place, the receiving bulb $B$ is just full and the discharging bulb $A$ is empty. Now if liquid flows under its own weight the following expression holds, which is Poiseuille's law applied to the present case (Reference 2):

$$\mu = \frac{\pi r^4 g \rho}{8 l_e Q} \frac{2h_r}{\log_e\left(\frac{h_o+h_r}{h_o-h_r}\right)} t \tag{1}$$

where

$\mu$ = the absolute viscosity in poises.
$t$ = the time of discharge in seconds.
$Q$ = the volume of the discharging bulb in cm.³
$h_r$ = the height of $B$ or of $A$ (see fig. 1) in cm.
$h_o$ = the vertical distance between the centers of the bulbs $A$ and $B$ in cm.
$l_e$ = the effective length of the capillary tube in cm.
$r$ = the radius of the capillary tube in cm.
$\rho$ = the density of the liquid in grams per cm.³
$g$ = the acceleration of gravity in cm. per sec.²

In practice deviations are made from the viscometer just discussed, particularly in the design of the bulbs. The one which is adopted for the present work is shown in Figure 2. The discharging bulb is a combination of cone frustrums and a cylinder, while the receiving bulb is a cylinder which has the same radius as the cylinder in the discharging bulb. The volume of the discharging bulb $A$ is included between the marks $M_2$ and $M_1$, which will be spoken of as the timing marks.

In order to compensate for the slight deviation from the ideal case, equation (1) is modified as follows:

$$\mu = \frac{\pi r^4 g \rho}{8 l_e Q} \frac{2h_r t}{\log_e\left(\frac{h_o+h_r+C}{h_o-h_1+C}\right)} \tag{2}$$

or putting

$$A_0 = \frac{\pi r^4 g}{8 l_e Q}$$

$$\mu = \rho A_0 \frac{2h_r t}{\log_e\left(\frac{h_o+h_r+C}{h_o-h_r+C}\right)} \tag{3}$$

Here all of the quantities except $h_o$, $h_r$, and $C$ have the same meaning as in equation (1). $C$ is to be determined experimentally, while $h_o$ and $h_r$ are defined in the follwoing paragraph.

The quantity $C$ may involve a constant term and an expression dependent on $h_0$. The latter would be small in comparison with the constant term and therefore can be neglected.

It will be supposed that when the liquid in the viscometer is in equilibrium under its own weight its free surface in the receiving tube $B$ occupies the mark $m_5$. (See fig. 2.) Such marks will be spoken of as the filling mark of the viscometer. When the liquid is forced to the level of the timing mark $M_1$ of the discharging bulb, the free surface of the liquid in the receiving tube will descend to a level $n_1$. When the liquid is next forced to the timing mark $M_2$, the corresponding level in the receiving tube will be $n_2$. Denoting by $H_1$ and $H_2$ the vertical distances



FIG. 1                    FIG. 2

of $M_1$ and $M_2$ from $m_5$ and by $h_2$ and $h_1$ the vertical distances of $n_2$ and $n_1$ from $m_5$, the definitions of $h_0$ and $h_r$ are

$$h_r = h_2 - h_1$$

$$h_0 = \frac{H_1 + H_2 + h_1 + h_2}{2} \tag{4}$$

Equations (2) and (3) hold only for slow rates of flow. If the velocity is fast, the following equation applies:

$$\mu = \rho \left[ A_0 \frac{2 h_r t}{\log_e \left( \frac{h_0 + h_r + C}{h_0 - h_r + C} \right)} - \frac{B}{t} \right] \tag{5}$$

where

$$B = \frac{mQ}{8 \pi l_e} \tag{6}$$

The term $\frac{B \rho}{t}$ is known as the inertia term. The quantity $m$ which appears in the definition of $B$ is a constant quantity for a given viscometer; its value may vary from 1 to 2, depending on the configuration of the ends. (Reference 4.)

A relation proposed by Bond (Reference 3) is that equation (3) applies when

$$\frac{Q'\rho}{r\mu} = \frac{Q}{r}\frac{\rho}{\mu t} = \frac{Q}{r}B_c \lessgtr 16 \qquad (7)$$

and that equation (5) should be used for values in excess of 16. $Q'$ is the volume rate of flow of the liquid, which is $Q/t$ for all practical purposes. $B_c$ may be called Bond's criterion. This relation was shown to hold only for short-length, thick-walled, and square-edged capillaries. In capillaries with trumpet-shaped ends the quantity $\frac{Q}{r}B_c$ is larger than 16. (Reference 4.)

It should be noted that multiplying the quantity $\frac{Q}{r}B_c$ by $\frac{2}{\pi}$ gives the expression for the more familiar Reynolds Number; thus the criterion 16 is equivalent to a Reynolds Number of $\frac{32}{\pi}$.

The viscometers used during measurements were four in number. They were alike in design, except for the radii of the bores, and conform to the data given in the sketch in Figure 2. In each viscometer there were five filling marks, allowing five different rates of flow. Generally it is the lowest one—that is, $m_5$, which was used during the tests. However, use was made of the other filling marks in the calibration of the viscometers. The proportions of the discharging bulb were designed for the smallest error in drainage (Reference 5), which requires that the height of the conical portion is at least equal to the height of the cylindrical portion. The distance between the timing marks was nearly 3.5 cm., while the distances between two consecutive filling marks were nearly 3 cm. Approximate essential dimensions, such as the radii of capillaries $r$, the volumes of discharging bulbs $Q$, the length of capillaries when the trumpetlike ends are excluded, $l_2$, and when they are included, $l_1$, are given in Table I. The dimension $r$ was found by weighing the mercury contained in a measured length of the capillary tube and $Q$ by weighing mercury both at a temperature of 23° C.

TABLE I

| Viscometer | Radius of capillary $r$ | Volume of bulb $Q$ | Length of capillary | |
|---|---|---|---|---|
| | | | $l_2$ | $l_1$ |
| | $Cm.$ | $Cm.^3$ | $Cm.$ | $Cm.$ |
| $V_1$-------------- | 0. 0519 | 6. 37 | 19. 6 | 20. 2 |
| $V_2$-------------- | . 0641 | 5. 84 | 19. 6 | 20. 2 |
| $V_3$-------------- | . 1085 | 6. 81 | 19. 5 | 20. 2 |
| $V_4$-------------- | . 1283 | 6. 28 | 19. 3 | 20. 0 |

CALIBRATION OF VISCOMETERS

Equation (5) is the working formula of the viscometers which shall be denoted by $V_1$, $V_2$, $V_3$, and $V_4$. There are three constants to be determined by calibration; these constants are $A_0$, $B$, and $C$. The equation (5) can be made linear in these constants if $C$ is sufficiently small with respect to $h_0 - h_r$. It will be assumed that this is the case. It can be shown that

$$\log_e \frac{h_0 + h_r + C}{h_0 - h_r + C} = \log_e \left(\frac{h_0 + h_r}{h_0 - h_r}\right) - \frac{2h_r C}{h_0^2 - h_r^2} + \frac{2h_r}{h_0^2 - h_r^2} \cdot \frac{h_0}{h_0^2 - h_r^2} C^2$$

or defining

$$N = \log_e \frac{h_0 + h_r}{h_0 - h_r}$$

$$M = \frac{2h_r}{h_0^2 - h_r^2}$$

$$L = \frac{h_0}{h_0^2 - h_r^2}$$

$$\left(\log_e \frac{h_0 + h_r + C}{h - h_r + C}\right)^{-1} = \frac{1}{N} + \frac{M}{N^2}C - \frac{M}{N^2}\left(L - \frac{M}{N}\right)C^2 \qquad (8)$$

TABLE II

DATA FOR EVALUATING THE EFFECTIVE HEAD

| Viscometer | Filling mark | $h_o$ | $h_6$ | $\frac{1}{N}$ | $M$ | $\frac{M}{N^2}$ | $L$ |
|---|---|---|---|---|---|---|---|
| | | Cm. | Cm. | | | | |
| $V_1$ | $m_1$ | | 6.52 | 1.840 | 0.0881 | 0.296 | 0.1650 |
| | $m_2$ | | 9.59 | 2.743 | .0387 | .292 | .1078 |
| | $m_3$ | 1.73 | 12.59 | 3.621 | .0223 | .292 | .0810 |
| | $m_4$ | | 15.59 | 4.498 | .0143 | .292 | .0649 |
| | $m_5$ | | 18.60 | 5.362 | .0101 | .290 | .0542 |
| $V_2$ | $m_1$ | | 6.94 | 2.102 | .0711 | .314 | .1524 |
| | $m_2$ | | 9.98 | 3.051 | .0334 | .311 | .1029 |
| | $m_3$ | 1.62 | 12.97 | 3.976 | .0196 | .309 | .0738 |
| | $m_4$ | | 16.01 | 4.929 | .0128 | .309 | .0631 |
| | $m_5$ | | 19.05 | 5.862 | .0090 | .301 | .0529 |
| $V_3$ | $m_1$ | | 6.98 | 1.771 | .0853 | .267 | .1550 |
| | $m_2$ | | 9.99 | 2.569 | .0399 | .263 | .1039 |
| | $m_3$ | 1.92 | 12.98 | 3.358 | .0233 | .263 | .0788 |
| | $m_4$ | | 15.98 | 4.142 | .0152 | .262 | .0635 |
| | $m_5$ | | 19.04 | 4.948 | .0107 | .262 | .0531 |
| $V_4$ | $m_1$ | | 6.55 | 1.594 | .1022 | .260 | .1682 |
| | $m_2$ | | 9.63 | 2.384 | .0448 | .255 | .1085 |
| | $m_3$ | 1.99 | 12.61 | 3.141 | .0257 | .253 | .0814 |
| | $m_4$ | | 15.62 | 3.903 | .0166 | .253 | .0651 |
| | $m_5$ | | 18.56 | 4.649 | .0117 | .253 | .0545 |

The values of $\frac{1}{N}\frac{M}{N^2}$, $M$ and $L$ for each filling mark of the four viscometers appear in Table II. It is evident from this table that the term containing $C^2$ in equation (8) is insignificant. Accordingly (5) becomes

$$\frac{\mu}{\rho}=\nu=A'_o\left[\frac{1}{N}+\frac{M}{N^2}C\right]t-\frac{B}{t} \tag{9}$$

where $A'_o=2h_rA_o$ and $\nu$ represent the kinematic viscosity. By definition kinematic viscosity is the ratio of the viscosity in absolute units to the density.

In equation (9) the quantities $A'_o$, $C$, and $B$ are constants for each viscometer, independent of the filling marks. The term $2h_rg\rho\left(\frac{1}{N}+\frac{M}{N^2}C\right)$ is the effective pressure of which $2h_rg\rho$ is constant for each viscometer and included in with $A'_o$, while $\frac{1}{N}+\frac{M}{N^2}C$ varies for each filling mark.

Since each viscometer has five filling marks, one standard liquid would be sufficient to determine the three constants, $A'_o$, $C$, and $B$, providing that $t$ could be measured with sufficient accuracy. In the determination of these constants at least three appropriate standard liquids were used for each viscometer. The viscosities of these liquids at $+25°$ C., except that of water, were furnished by the friction and lubrication section of the Bureau of Standards. The majority of them were mineral oils, and their viscosity values perhaps had undergone slight variation before use was made of them as calibrating liquids. During the calibrations no particular effort was exerted to observe the flows exactly at 25° C. as long as the temperature of the liquid remained in the vicinity of 25° C. By previously determining the thermal coefficient of each standard liquid all the observed values of the time of flow were reduced to that at 25° C. These reduced values of $t_{25}$ obtained for the various viscometers are given in Tables IV, V, VI, and VII. When the known values of $\nu_{25}$ and the observed values of $t_{25}$ were substituted in equation (9), the substitutions gave concordant values for $A'_o$ and $C$ on the supposition that $m=1.1$. The values are given below:

All of the constants in equation (9) are now known for the viscometers.

| Viscometer | $C$ | $A'_o$ |
|---|---|---|
| | Cm. | |
| $V_1$ | -0.09 | $0.776\times10^{-4}$ |
| $V_2$ | -.30 | $1.891\times10^{-4}$ |
| $V_3$ | -.24 | $1.540\times10^{-4}$ |
| $V_4$ | -.34 | $3.557\times10^{-4}$ |

The following procedure was adopted in order to study the concordance of the observations given in Figures 5 to 7 and the values of the constants. This will place the results of the calibration in final form for use in measuring viscosities. Define

$$\nu_1 = A_1 t \qquad (10)$$

where

$$A_1 = A'_0 \left( \frac{1}{N} + \frac{M}{N^2} C \right)$$

The values of $A_1$ for each filling mark in each viscometer are given in Table III. Equation (10) gives the value of the kinematic viscosity $\nu_1$ neglecting the term $\frac{B}{t}$. Denoting by $\nu$ the correct value of the kinematic viscosity, the percentage error of the kinematic viscosities when formula (10) is used becomes $\frac{\nu_1 - \nu}{\nu} \times 10^2$. These errors for the calibration are given in Tables IV, V, VI, and VII and in Figures 3, 4, and 5. The ratio $\frac{1}{t\nu}$ is plotted against these errors. This procedure is substantially that proposed by Herschel (Reference 6), since $\frac{1}{t\nu}$ is proportional to Reynolds Number, which is in our notation $\frac{2Q}{\pi r \nu t}$.

### TABLE III

CALIBRATION OF VISCOMETERS $V_1$, $V_2$, $V_3$, AND $V_4$—VALUE OF $A_1$

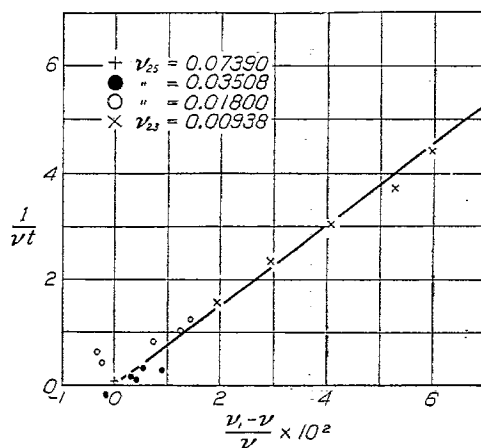| Viscometer | Filling mark | $A_1$ | Viscometer | Filling mark | $A_1$ |
|---|---|---|---|---|---|
| $V_1$ | $m_5$ | $4.141 \times 10^{-4}$ | $V_3$ | $m_5$ | $7.524 \times 10^{-3}$ |
|  | $m_4$ | $3.470 \times 10^{-4}$ |  | $m_4$ | $6.283 \times 10^{-3}$ |
|  | $m_3$ | $2.790 \times 10^{-4}$ |  | $m_3$ | $5.074 \times 10^{-3}$ |
|  | $m_2$ | $2.109 \times 10^{-4}$ |  | $m_2$ | $3.859 \times 10^{-3}$ |
|  | $m_1$ | $1.408 \times 10^{-4}$ |  | $m_1$ | $2.630 \times 10^{-3}$ |
| $V_2$ | $m_5$ | $1.091 \times 10^{-3}$ | $V_4$ | $m_5$ | $1.623 \times 10^{-2}$ |
|  | $m_4$ | $.9145 \times 10^{-3}$ |  | $m_4$ | $1.358 \times 10^{-2}$ |
|  | $m_3$ | $.7343 \times 10^{-3}$ |  | $m_3$ | $1.090 \times 10^{-2}$ |
|  | $m_2$ | $.5594 \times 10^{-3}$ |  | $m_2$ | $.8170 \times 10^{-2}$ |
|  | $m_1$ | $.3799 \times 10^{-3}$ |  | $m_1$ | $.5357 \times 10^{-2}$ |



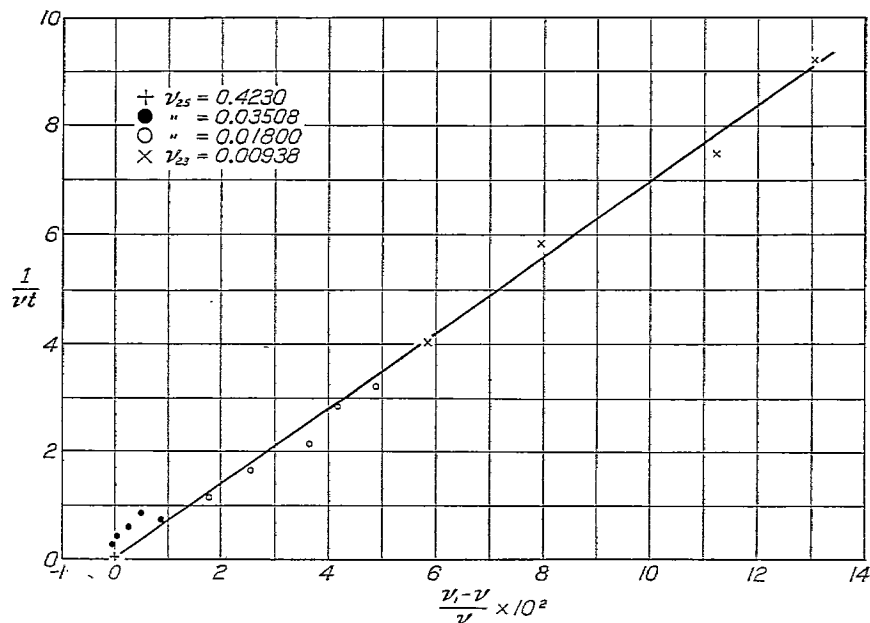FIG. 3.—Calibration of viscometer No. 1
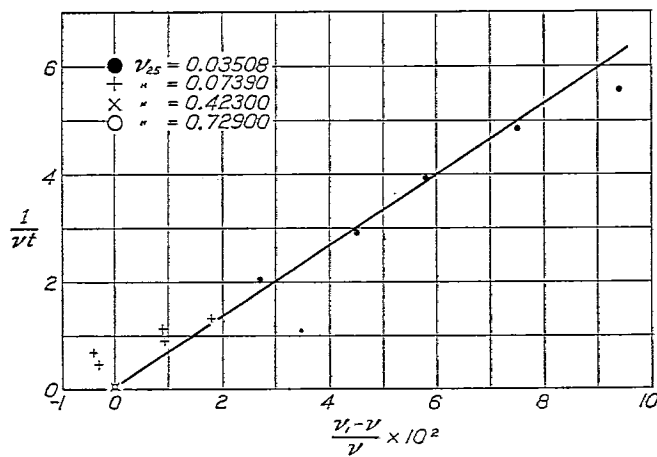
Fig. 4.—Calibration of viscometer No. 2



Fig. 5.—Calibration of viscometer No. 3

## TABLE IV

### CALIBRATION OF VISCOMETER $V_1$

| Kinematic viscosity of liquid | Filling mark | Time $t$ | Kinematic viscosity $\nu_1$ | Error $\frac{\nu_1-\nu}{\nu}\times10^2$ | $\frac{1}{t\nu}$ |
|---|---|---|---|---|---|
| | | *Seconds* | | | |
| $\nu_{25}=0.07390$ | $m_5$ | 178.8 | 0.07404 | _____ | 0.07 |
| $\nu_{25}=0.0350S$ | $m_5$ | 85.2 | .03528 | 0.56 | .33 |
| | $m_4$ | 102.0 | .03540 | .91 | .28 |
| | $m_3$ | 126.9 | .03540 | .91 | .22 |
| | $m_2$ | 166.9 | .03519 | .31 | .17 |
| | $m_1$ | 250.3 | .03523 | .42 | .11 |
| $\nu_{25}=0.01800$ | $m_5$ | 44.1 | .01826 | 1.44 | 1.25 |
| | $m_4$ | 52.5 | .01822 | 1.22 | 1.05 |
| | $m_3$ | 65.0 | .01813 | .72 | .85 |
| | $m_2$ | 85.1 | .01794 | —.32 | .65 |
| | $m_1$ | 127.6 | .01796 | —.22 | .43 |
| $\nu_{23}=0.00938$ | $m_5$ | 24.0 | .00994 | 5.96 | 4.45 |
| | $m_4$ | 28.4 | .009S6 | 5.24 | 3.77 |
| | $m_3$ | 35.0 | .00976 | 4.05 | 3.07 |
| | $m_2$ | 45.8 | .00966 | 2.9S | 2.35 |
| | $m_1$ | 67.9 | .00956 | 1.92 | 1.5S |

## TABLE V

CALIBRATION OF VISCOMETER $V_2$

| Kinematic viscosity of liquid | Filling mark | Time $t$ | Kinematic viscosity $\nu_1$ | Error $\frac{\nu_1-\nu}{\nu}\times10^2$ | $\frac{1}{t\,\nu}$ |
|---|---|---|---|---|---|
| $\nu_{25}=0.4230$ | $m_5$ | *Seconds* 386. 6 | 0. 4229 | .0 | 0. 01 |
| $\nu_{25}=0.03508$ | $m_5$ | 32. 3 | . 03526 | . 50 | . 88 |
| | $m_4$ | 38. 7 | . 03539 | . 89 | . 74 |
| | $m_3$ | 47. 9 | . 03517 | . 26 | . 60 |
| | $m_2$ | 62. 7 | . 03507 | . 03 | . 45 |
| | $m_1$ | 92. 3 | . 03506 | . 06 | . 31 |
| $\nu_{25}=0.01800$ | $m_5$ | 17. 3 | . 01888 | 4. 89 | 3. 20 |
| | $m_4$ | 20. 5 | . 01875 | 4. 17 | 2. 72 |
| | $m_3$ | 25. 4 | . 01865 | 3. 61 | 2. 16 |
| | $m_2$ | 33. 0 | . 01846 | 2. 55 | 1. 67 |
| | $m_1$ | 48. 2 | . 01831 | 1. 72 | 1. 16 |
| $\nu_{23}=0.00938$ | $m_5$ | 9. 9 | . 01080 | 15. 02 | 10. 80 |
| | $m_4$ | 11. 6 | . 01061 | 13. 02 | 9. 22 |
| | $m_3$ | 14. 2 | . 01043 | 11. 20 | 7. 50 |
| | $m_2$ | 18. 1 | . 01012 | 7. 98 | 5. 89 |
| | $m_1$ | 26. 1 | . 00992 | 5. 81 | 4. 07 |

## TABLE VI

CALIBRATION OF VISCOMETER $V_1$

| Kinematic viscosity of liquid | Filling mark | Time $t$ | Kinematic viscosity $\nu_1$ | Error $\frac{\nu_1-\nu}{\nu}\times10^2$ | $\frac{1}{t\,\nu}$ |
|---|---|---|---|---|---|
| $\nu_{25}=0.7290$ | $m_5$ | *Seconds* 96. 9 | 0. 7290 | 0. 00 | 0. 01 |
| $\nu_{25}=0.4230$ | $m_5$ | 56. 2 | . 4232 | . 00 | . 04 |
| | $m_4$ | 67. 3 | . 4231 | . 00 | . 03 |
| | $m_3$ | 83. 3 | . 4230 | . 00 | . 03 |
| | $m_2$ | 109. 6 | . 4229 | . 00 | . 02 |
| | $m_1$ | 161. 2 | . 4230 | . 00 | . 01 |
| $\nu_{25}=0.07390$ | $m_5$ | 10. 0 | . 07525 | 1. 83 | 1. 35 |
| | $m_4$ | 11. 9 | . 07454 | . 86 | 1. 15 |
| | $m_3$ | 14. 7 | . 07458 | . 92 | . 92 |
| | $m_2$ | 19. 1 | . 07361 | —. 39 | . 71 |
| | $m_1$ | 28. 0 | . 07370 | —. 27 | . 48 |
| $\nu_{25}=0.03508$ | $m_5$ | 5. 1 | . 03838 | 9. 4 | 5. 53 |
| | $m_4$ | 6. 0 | . 03771 | 7. 5 | 4. 84 |
| | $m_3$ | 7. 3 | . 03704 | 5. 8 | 3. 91 |
| | $m_2$ | 9. 5 | . 03666 | 4. 5 | 3. 00 |
| | $m_1$ | 13. 7 | . 03604 | 2. 7 | 2. 05 |

TABLE VII

CALIBRATION OF VISCOMETER $V_4$

| Kinematic viscosity of liquid | Filling mark | Time $t$ | Kinematic viscosity $\nu_1$ | Error $\frac{\nu_1 - \nu}{\nu} \times 10^2$ | $\frac{1}{t\nu}$ |
|---|---|---|---|---|---|
| $\nu_{25} = 0.729$ | $m_5$ | *Seconds* 44.8 | 0.727 | −0.28 | 0.030 |
| | $m_4$ | 53.9 | .736 | +.96 | .025 |
| | $m_3$ | 66.3 | .720 | −1.25 | .021 |
| | $m_2$ | 89.1 | .728 | −.14 | .016 |
| | $m_1$ | 135.5 | .726 | −.42 | .009 |
| $\nu_{25} = 0.783$ | $m_5$ | 48.0 | .780 | −.38 | .027 |
| | $m_4$ | 57.4 | .779 | −.51 | .022 |
| | $m_3$ | 71.6 | .781 | −.26 | .018 |
| | $m_2$ | 95.2 | .778 | −.64 | .013 |
| | $m_1$ | 145.5 | .780 | −.38 | .009 |
| $\nu_{25} = 0.423$ | $m_5$ | 26.2 | .425 | +.47 | .0902 |
| | $m_4$ | 31.3 | .425 | +.47 | .076 |
| | $m_3$ | 39.1 | .426 | +.71 | .062 |
| | $m_2$ | 52.0 | .425 | +.47 | .045 |
| | $m_1$ | 79.3 | .425 | +.47 | .028 |

The graphs which are straight lines were made to pass through the origin. This procedure of permitting the straight lines to pass through the origin is quantitatively contrary to Bond's criterion. The criterion was ignored purposely in this respect, as the errors of the calibration were sufficiently large to eclipse its importance. For from equation (7) Bond's criterion $B_c$ for the viscometers $V_1$, $V_2$, $V_3$, and $V_4$ becomes $\frac{1}{t\nu} = B_c = 0.13, 0.17, 0.26$, and $0.33$, respectively. As all the observations made with $V_4$ were below the value of Bond's criterion, the data of Table VII are not plotted.

Now it remains to be seen if the assumed value of $m = 1.1$ during the determinations of $A'_0$ and $C$ is in accordance with the graphs in Figures 2, 3, and 4. As

$$\nu_1 = A_1 t$$

and

$$\nu = A_1 t - \frac{B}{t}$$

then

$$\nu_1 - \nu = \frac{B}{t}$$

or

$$\frac{\nu_1 - \nu}{\nu} = \frac{B}{t\nu} \tag{10A}$$

This gives $B \times 10^2 = \dfrac{1}{\tan \alpha}$

where $\tan \alpha$ is the slope of the straight lines in Figures 3, 4, and 5. (See References 6, 7, and 8.) Finally, if $l_e$ is supposed not to differ much from $l_2$, expression (6) gives

$$m = \frac{8\pi l_2}{Q} \times \frac{10^{-2}}{\tan \alpha} \tag{11}$$

Calculations from the data of the calibration graphs gave $m = 1.00, 1.18$, and $1.11$ for viscometers $V_1$, $V_2$, and $V_3$, respectively. This shows that the values of $B$ when determined from the calibration graph would have an accuracy of 10 per cent. On the other hand, the values of $A_1$ which are given in Table III have an accuracy of 0.5 per cent at least as judged from the individual observations of the calibration.

## MEASUREMENT OF KINEMATIC VISCOSITY

The procedure in evaluating $\nu$ for some particular liquid consisted in determining $\nu_1$ by means of equation (10) after the time of discharge $t$ was observed. The values of $\nu_1$ against temperature are given in graphs, as described later. In order to determine the values of $\nu$ given in this report, the quantity $\frac{1}{t\nu_1}$ was computed and the error $e$ read from the calibration graph for the viscometer used. (Fig. 3 to 5.) The value of $\nu$ then becomes

$$\nu = \nu_1 (1 - e) \tag{12}$$

### DENSITY AND THERMAL COEFFICIENT OF CUBICAL EXPANSION

When $t_\theta$ is known for a given liquid, formula (10) gives the kinematic viscosity $\nu_\theta$ for the temperature $\theta$. To obtain the absolute viscosity $\mu_\theta$ from the known value of $\nu_\theta$ the density $\rho_\theta$ at the temperature $\theta$ must be known. To obtain the density $\rho_\theta$ in the temperature range $+30°$ C. to $-20°$ C. with an accuracy adequate for the purpose of the present paper, it suffices to measure $\rho_{30}$ and the average thermal coefficient of cubical expansion $\alpha$ for the range $+30°$ C. to $0°$ C. The definition of $\alpha$ is

$$\alpha = \frac{V_{30} - V_0}{30 \times V_{30}} \tag{13}$$

where $V_0$ represents the volume of a given mass of liquid at the temperature $0°$ C. Then the density $\rho_\theta$ is given by

$$\rho_\theta = \rho_{30} [1 + \alpha (30 - \theta)] \tag{14}$$

The determinations of the density $\rho_{30}$ and the coefficient of expansion were carried out in the well-known orthodox manner; no other comment need be made except the statement that $\rho_{30}$ is determined with an accuracy of 0.2 per cent and $\alpha$ with an accuracy of 3 per cent. The results are given in Table VIII, in which $\alpha$ is the coefficient for temperatures in degrees centigrade.

The values of the change in density per degree centigrade for petroleum oils against specific gravity based on the National Standard Petroleum Oil Tables (Reference 10) are given in convenient form in Reference 9. The data in Table VIII are in agreement within the experimental error.

### TEMPERATURE CONTROL

The four viscometers $V_1$, $V_2$, $V_3$, and $V_4$ arranged in a plane were held parallel to each other by means of two pairs of wooden clamps. To insure fixity of position, a single arm of each clamp was rigidly attached to the two vertical sides of a rectangular brass frame. A heavy rectangular plate was attached to the upper edge of the frame so as to be perpendicular to the vertical plane containing the axis of the viscometers. This plate will be spoken of as the holding plate. When the holding plate is kept horizontal, the axis of the capillaries are in a vertical position. The description so far given relates to the viscometer assembly. Next consider the bath of the viscometers. It consisted of a rectangular glass vessel of over-all dimensions 13 by 22 by 31 centimeters, placed in a wooden frame. The frame embraced the bottom and the two narrow sides of the glass vessel. It also provided a smooth rectangular brim to the glass vessel. The liquid of the bath was a mixture of water and alcohol.

When the viscometer assembly was immersed in the bath, the holding plate rested evenly on the wooden brim of the vessel. Thus if the brim was horizontal, the axis of capillaries would be vertically submerged in the bath. Eight holes in the holding plate allowed the ends of the viscometers to protrude. Two thermometers were employed to measure the temperature of the bath in the lower and upper portion of the vessel. An agitator in the form of a small propeller was placed in the bath for the equalization of temperature.

The viscometer assembly and the bath together constituted the viscometer box of the tests.

In order to control the temperature of the viscometer bath, the viscometer box was placed in a temperature chamber in which the reduction of temperature was effected by the use of an ammonia refrigeration system. An electric heater was the source of heat in the chamber.

Because of the large mass of liquid in the bath, it generally took seven hours to reduce the temperature of the bath from an initial value of 0° C. to −18° C., when the temperature of the chamber was held continually at its lowest temperature, that is, −23° C. This property of the apparatus is the factor limiting the lowest values of the temperature realized during the tests.

### ERRORS OF THE MEASUREMENTS

Three main sources of experimental error exist; calibration, observation of time of flow, and determination of temperature. As remarked previously, errors of calibration did not appear to be greater than 0.5 per cent. The errors of measurement of time need be considered only in connection with the observation of small values of the time, $t_\theta$. In the tests the smallest values of $t_\theta$ were in the neighborhood of 10 seconds. To reduce the error, measurements of time for a given temperature were made three times or more and the average noted. Quite generally in runs in which $t_{30}$ was approximately 10 seconds individual observations of $t_\theta$ did not differ more than 0.1 second from the average. It follows that viscosity determinations for the lowest value of $t_{30}$ are open to an error of not more than 1 per cent. Temperature errors may arise in two ways: First, that during a flow measurement the temperature of the bath had a temperature gradient along the capillaries; secondly, that the average temperature of the bath was not the same in the beginning of the flow as at the end. However, the temperature error was minimized by taking the averages of the average temperature of the bath at the beginning and at the end of the flow period. For most of the observations the temperature errors were not more than ±0.5° C.

## RESULTS OF INVESTIGATION

### SAMPLES

The solutions and liquids may be classified as follows:

(1) Solutions of animal and vegetable oils in xylene. These were poppy-seed oil, two samples of neat's-foot oil, castor oil, and linseed oil.

(2) Solutions of mineral oil in xylene. These were Squibbs' petrolatum of naphthene base and transformer oil.

(3) Glycerin solutions in ethyl alcohol and in mixture of 50–50 ethyl alcohol and water.

(4) Mixtures of normal butyl alcohol with methyl alcohol.

(5) Individual liquids; kerosene, mineral spirits, xylene, recoil oil.

The following information is available regarding the source, purity, and physical properties of the liquids:

### XYLENE

The following data were obtained from the label of the container: Chemical composition, $C_6H_4(CH_3)_2$; molecular weight, 106.12; specific gravity, 0.86; boiling point, 138°–139° C. It was found that the freezing point of the sample was below −35° C. Thorpe and Rodger's values of the viscosity for meta xylol were obtained from the Smithsonian Physical Tables (1920).

| Temperature (°C.) | Viscosity (poise) |
|---|---|
| 0 | 0.00806 |
| 10 | .00702 |
| 20 | .00620 |
| 30 | .00552 |

These values are about 2 per cent greater than the values found for the sample.

Xylene has been extensively used as the liquid for bubble levels for use on aircraft sextants and other instruments. Owing to its relatively low viscosity, low rate of change with temperature, and to the fact that the temperature at the melting point is below ordinary conditions of aircraft use, xylene is well adapted for use as a solvent. In this way solutions may be obtained which meet a variety of viscosity requirements.

## POPPY-SEED OIL

A sample was purchased without specifications from a wholesale dealer. This had a density at 15° C. of 0.92 gram per cm.³ compared with a value of 0.924 to 0.926 given in the International Critical Tables (Vol. II, 1927). The sample was found to have a cloud point of +7° C. and to freeze at −16° C.; the tables give −16° to −18° C. as the freezing point. The coefficient of viscosity of the sample was found to be 0.797 poise at 15.5° C. as compared with the value 0.789 poise given in the International Critical Tables.

The index of refraction of the sample was determined at two temperatures by the optical-instrument section of the Bureau of Standards. The values are given below, including also data on solutions of poppy-seed oil in xylene.

| Poppy-seed oil (per cent, by weight) | Xylene (per cent, by weight) | Index of refraction, $N_D$ | | Temperature coefficient of index per degree C. |
|---|---|---|---|---|
| | | 10.0° C. | 30.0° C. | |
| 100 | 0 | 1.4806 | 1.4731 | −0.00038 |
| 75 | 25 | 1.4862 | 1.4779 | −.00042 |
| 50 | 50 | 1.4914 | 1.4825 | −.00045 |
| 25 | 75 | 1.4964 | 1.4867 | −.00049 |

Poppy-seed oil and xylene solutions have found some use in dashpots of special aircraft instruments. As poppy-seed oil is a semidrying oil it requires frequent replacement.

## NEAT'S-FOOT OIL

Two samples of neat's-foot oil were furnished through the courtesy of the research department of Armour & Co. The samples are a "wintered" product and are known respectively as 20° and 30° F. (−6.7° C. and −1.1° C.) cold or flow test neat's-foot oil. The 20° F sample became cloudy at −14° C. and froze at about −16.5° C. The 30° F. sample became cloudy at −5° C. and froze at a temperature of −8° C.

The International Critical Tables (Vol. II, 1927) give 0.987 to 1.13 poises for the value of the viscosity at 15.5° C., compared with 1.15 poises for the 30° F. sample and slightly less for the 20° F. sample.

Neat's-foot oil is nonoxidizing in character, which is a desirable property in the selection of a liquid for use in an open dashpot.

## CASTOR OIL

The sample has not been analyzed for purity. The International Critical Tables (Vol. II, 1927) state that castor oil becomes turbid at −12° C. and solid at −17° C. to −18° C. The sample commenced freezing at −15° C. and was solid at −17° C. The density and viscosity are as follows according to the above reference:

| Temperature (° C.) | Density (grams/cm.³) | Viscosity (poises) |
|---|---|---|
| 5 | 0.9707 | 37.60 |
| 10 | .9672 | 24.18 |
| 20 | .9603 | 9.86 |
| 25 | .9569 | 6.51 |
| 30 | .9534 | 4.51 |

A comparison of the above viscosity data with that for the sample (given later) shows good agreement as regards the change in viscosity with temperature, but the viscosity of the sample is substantially 8 per cent greater.

## LINSEED OIL

The sample was raw linseed oil, purchased under United States Government Master Specification for Raw Linseed Oil. This specification is published as Bureau of Standards Circular No. 82 (third edition, issued April 26, 1927). The density of the sample is 0.932 gram per

cm.³ at 15.5° C., which is within the specific-gravity tolerance of 0.9300 to 0.935 (15:5°/15:5° C.), which is equivalent to the density interval 0.929 to 0.934 gram per cm.³ The cloud point of the sample is approximately −7° C., freezing in the interval −14° to −17° C. The International Critical Tables (Vol. II, 1927) give the freezing point as −19° to −27° C. and the viscosity at 15.5° C. as 0.55 poises, while the value found for the sample is 0.585 poises.

### SQUIBBS' MINERAL OIL (LIQUID PETROLATUM)

This oil is a commercial product sold for medical use. The label on the bottle states that the oil is refined from California crude petroleum (naphthene base) and that it is free from paraffin, inorganic matter, and organic sulphur compounds. The sample froze at a temperature of −21° to −22° C.

### TRANSFORMER OIL

This oil is a sample of transformer oil which, contrary to expectations, froze in the temperature range −32° to −33° C.

### GLYCERINE

If the density of the sample is taken as the criterion, then the glycerine of our tests contained 4 per cent water, for, according to the International Critical Tables (Vol. III) the density of 96 per cent aqueous solution of glycerol at 25° C. is 1.2477, and for the same temperature the density of the sample was found to be 1.2480 grams per cm.³ Shöttner's values of the viscosity of glycerol are given in the Physico-Chemical Tables (Vol. II). According to these tables the viscosity of glycerol at 26.5° C. is 4.939 poises. Our determination of the same quantity gives 3.909 poises. The considerable difference between these two values is due to the water content in our sample.

### ETHYL ALCOHOL

The sample initially contained less than 2 per cent of water. It should be noted that measurements on a given sample of liquid extended over a period of three days. In the case of hygroscopic liquids and solutions such as alcohol and glycerine some change in water content may have occurred.

### N. BUTYL ALCOHOL

The sample was purchased from the Eastman Kodak Laboratores. Two grades of the substance are available; one is known as technical and has a boiling point of 114° to 118° C.; the other is a purer product and has a boiling point of 116° to 118° C. The latter product was used in the tests.

The International Critical Tables (Vol. I, 1926) give the melting point as −89.8° C. and the density as 0.810 gram per cm.³ at 20° C. The values of the viscosity found by Thorpe and Rodger (1894) as given in Landolt-Börnstein Tables (Vol. I, 1923) agree within 1 per cent with the values found for the sample. Thorpe and Rodger's values are:

| Temperature (° C.) | Viscosity (poise) |
|---|---|
| 0.27 | 0.0515 |
| 10.69 | .0380 |
| 21.83 | .0280 |

### METHYL ALCOHOL

The sample ($CH_3OH$) was of reagent quality. The manufacturer's label gave the following analysis:

| | Per cent | | Per cent |
|---|---|---|---|
| Acetone | 0 | Empyreuma | 0.001 |
| Aldehyde | 0 | Reaction to litmus | Neutral. |
| Chloroform | .01 | Reducing matter | 0 |

The International Critical Tables (Vol. I, 1926) give the melting point of methyl alcohol as −97.8° C. This alcohol when mixed with butyl alcohol has been proposed for use in rolling ball inclinometers.

## KEROSENE

The sample of kerosene had a density of 0.810 gram per cm.$^3$ at 30° C. and a freezing point of −15° C.

## MINERAL SPIRITS

This is a petroleum product also known as Varnolene, which is used as a paint thinner. It is generally used as the damping liquid in aircraft compasses. The sample is commercial, most likely a naphthene (asphalt) base. Its density is 0.769 gram per cm.$^3$ at 30° C. A specification of mineral spirits based on its use as a paint thinner is given in Bureau of Standards Circular No. 98.

The sample was found to remain clear when at a temperature of −35° C.

## RECOIL OIL

This sample conforms to the specification for recoil oil, light grade, given in United States Master Specification for Lubricants and Liquid Fuels, Bureau of Mines Technical Paper 323–B. It was found to remain unfrozen at a temperature of −30° C.

## 2. EXPERIMENTAL RESULTS

Figures 6 to 17, inclusive, show the effect of temperature on the kinematic viscosity of the various solutions and liquids. The concentrations of the solutions are noted in the figures and refer to weight.

The ordinates $n_\theta$ are the quantities defined by

$$n_\theta = \log_{10} \frac{t_\theta}{t_{30}} \tag{15}$$

where $t_\theta$ is the observed time of discharge at temperature $\theta$. Here $t_{30}$ was not observed generally but was evaluated from the observed $t_\theta$ in the temperature range 30° C. to 15° C. Thus the value of $t_{30}$ is more probable than that of a single observation made at 30° C.

Now, denote by $n'_\theta$ the quantity

$$n'_\theta = \log_{10} \frac{\nu_\theta}{\nu_{30}} \tag{16}$$

where $\nu_\theta$ is the kinematic viscosity at the temperature $\theta$. The quantity $n'$ will have the same value as $n_\theta$ if the inertia term is negligible in the flows of time $t_\theta$ and $t_{30}$. Strictly speaking, this was not the case, particularly for the measurements of $t_{30}$ made with the viscometer $V_1$. For many purposes the differences $n'_\theta - n_\theta$ being small can be neglected; with this restriction, then, the curves in Figures 6 to 17, inclusive, are representative of the temperature effect on kinematic viscosity.

The magnitudes of the differences $n'_\theta - n_\theta$ are always less than the value for $e$ given in the graphs, as is evident from equation (18) below, and therefore the values of $n_\theta$ are, on the whole, except for the data on Figures 15 and 16, within 1 per cent of $n'_\theta$. The data for the 50–50 butyl-methyl alcohol in Figure 15 shows an extreme error $e$ of 9.5 per cent. If the values of $n_\theta$ given in the graph are used to compute $\nu$, the result is in error 0.5 per cent at + 10° C. and 6.5 per cent at −15° C.

Assuming that the difference of $n'_\theta - n_\theta$ is negligible, the viscosity $\nu_\theta$ can be computed from the relation

$$n_\theta = \log \frac{\nu_\theta}{\nu_{30}}$$

in which $n_\theta$ is read from the proper graph (figs. 6 to 17), and $\nu_{30}$ is obtained from the tables given in the same figures.

The absolute viscosity $\mu_\theta$ is given by the relation

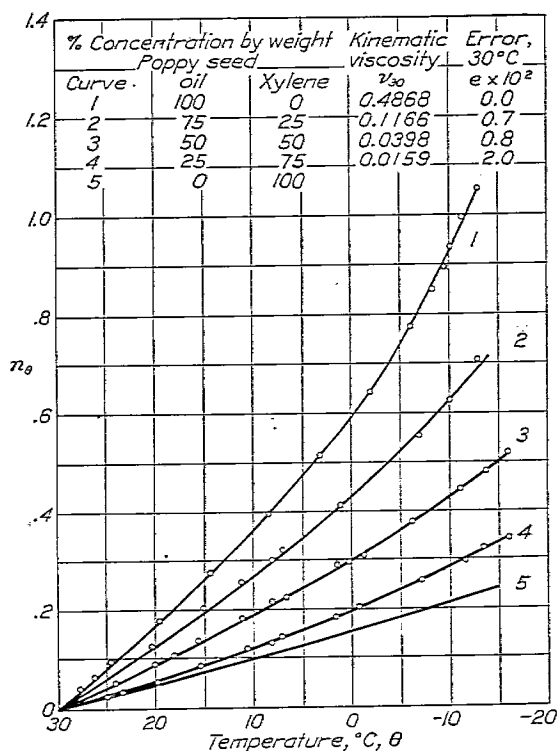$$\mu_\theta = \rho_\theta \nu_\theta$$

| Curve | % Concentration by weight Poppy seed | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| | oil | Xylene | $\nu_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.4868 | 0.0 |
| 2 | 75 | 25 | 0.1166 | 0.7 |
| 3 | 50 | 50 | 0.0398 | 0.8 |
| 4 | 25 | 75 | 0.0159 | 2.0 |
| 5 | 0 | 100 | | |

FIG. 6.—Change of kinematic viscosity with temperature. Poppy-seed oil and xylene



| Curve | % Concentration by weight Neatsfoot | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| | oil(20°F) | Xylene | $\nu_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.5875 | 0.0 |
| 2 | 75 | 25 | 0.1265 | 0.6 |
| 3 | 50 | 50 | 0.0406 | 0.8 |
| 4 | 25 | 75 | 0.0166 | 1.8 |
| 5 | 0 | 100 | | |
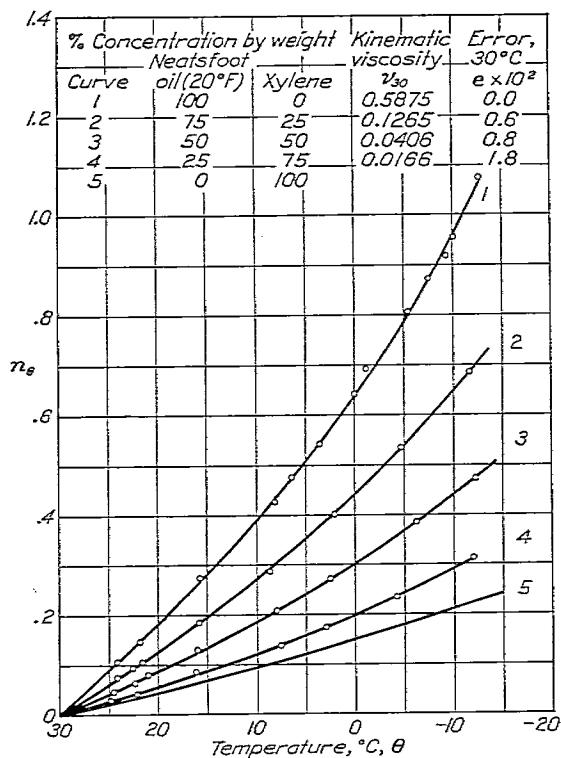
FIG. 7.—Change of kinematic viscosity with temperature. Neat's-foot oil and xylene



| Curve | % Concentration by weight Neatsfoot | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| | oil(30°F) | Xylene | $\nu_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.6529 | 0.0 |
| 2 | 75 | 25 | 0.1356 | 0.5 |
| 3 | 50 | 50 | 0.0410 | 0.8 |
| 4 | 25 | 75 | 0.0163 | 1.9 |
| 5 | 0 | 100 | | |
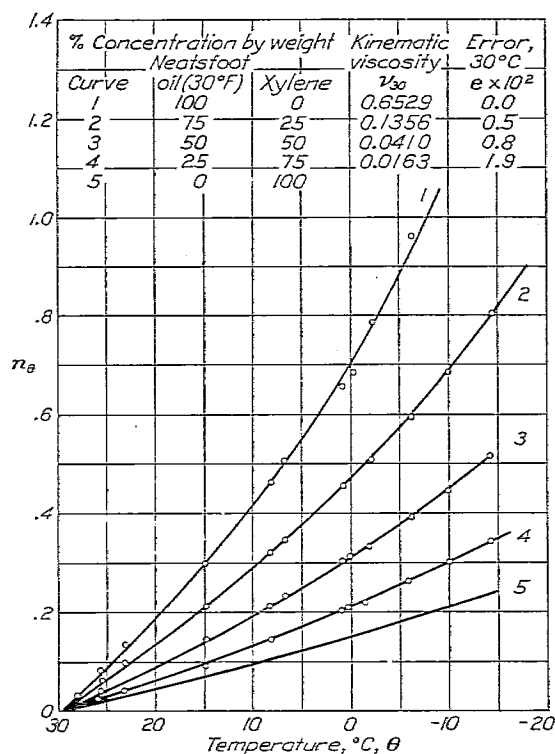
FIG. 8.—Change of kinematic viscosity with temperature. Neat's-foot oil (30° F.) and xylene



| Curve | % Concentration by weight Linseed | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| | oil | Xylene | $\nu_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.3701 | 0.0 |
| 2 | 75 | 25 | 0.0998 | 1.0 |
| 3 | 50 | 50 | 0.0356 | 1.1 |
| 4 | 25 | 75 | 0.0151 | 2.0 |
| 5 | 0 | 100 | | |
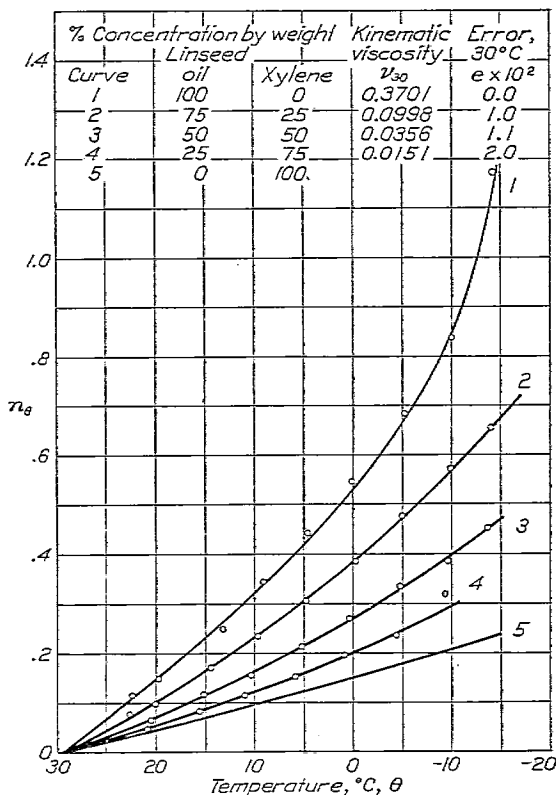
FIG. 9.—Change of kinematic viscosity with temperature. Linseed oil and xylene

| % Concentration by weight Castor | | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| Curve | oil | Xylene | $v_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 5.1250 | 0.0 |
| 2 | 75 | 25 | 0.4695 | 0.0 |
| 3 | 50 | 50 | 0.0784 | 0.1 |
| 4 | 25 | 75 | 0.0193 | 1.4 |
| 5 | 0 | 100 | | |
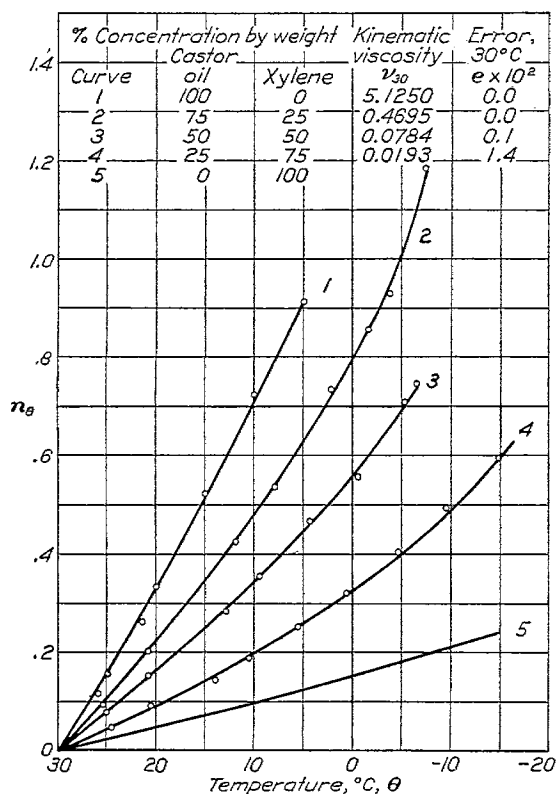
FIG. 10.—Change of kinematic viscosity with temperature. Castor oil and xylene

| % Concentration by weight Squibb's | | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| Curve | petrolatum | Xylene | $v_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.9100 | 0.0 |
| 2 | 75 | 25 | 0.0830 | 1.5 |
| 3 | 50 | 50 | 0.0249 | 2.1 |
| 4 | 25 | 75 | 0.0115 | 3.9 |
| 5 | 0 | 100 | | |
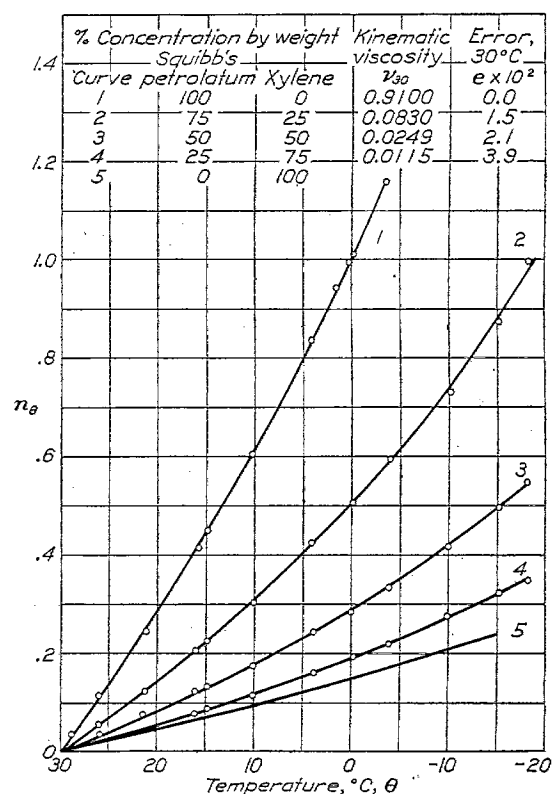
FIG. 11.—Change of kinematic viscosity with temperature. Squibb's petroleum and xylene

| % Concentration by weight Transformer | | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| Curve | oil | Xylene | $v_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 0.1119 | 0.8 |
| 2 | 80 | 20 | 0.0390 | 0.9 |
| 3 | 60 | 40 | 0.0197 | 1.3 |
| 4 | 0 | 100 | | |
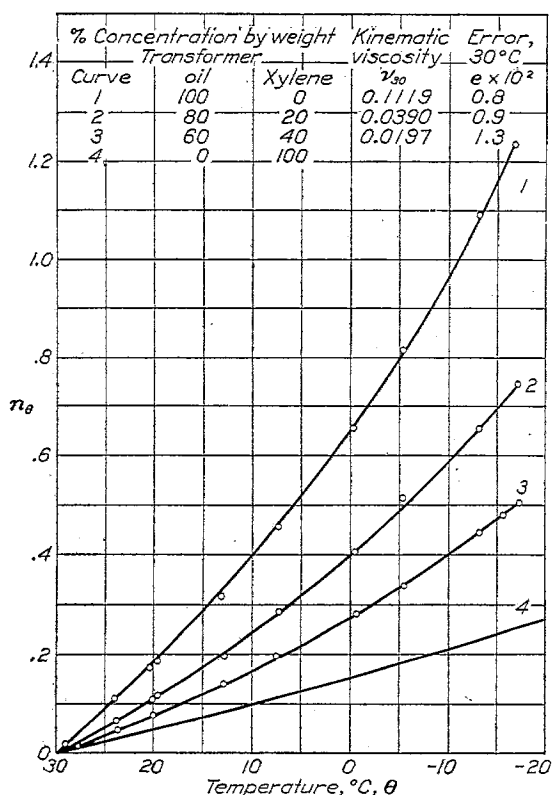
FIG. 12.—Change of kinematic viscosity with temperature. Transformer oil and xylene

| % Concentration by weight Ethyl | | | Kinematic viscosity | Error, 30°C |
|---|---|---|---|---|
| Curve | Glycerine | alcohol | $v_{30}$ | $e \times 10^2$ |
| 1 | 100 | 0 | 2.4120 | 0.0 |
| 2 | 75 | 25 | 0.3619 | 0.0 |
| 3 | 50 | 50 | 0.0960 | 1.1 |
| 4 | 25 | 75 | 0.0317 | 1.4 |
| 5 | 0 | 100 | | |

FIG. 13.—Change of kinematic viscosity with temperature. Glycerine and ethyl alcohol

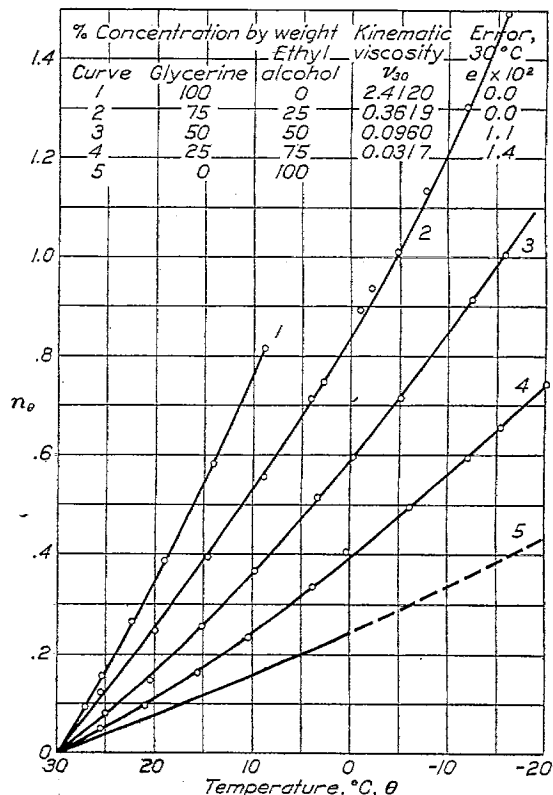For the purpose of determining the density at $\theta°$ C., the values of $\rho_{30}$ and the coefficient of volume expansion $\alpha$ are given in Table VIII. Equation (14) then serves to calculate the value of $\rho_\theta$.

In cases such that the difference $n'_\theta - n_\theta$ can not be ignored, the same procedure given above is followed except that $n'_\theta$ must be computed and used instead of $n_\theta$.

Let $e_\theta$ as given in equation (10A) represent the proportional error in $\nu_\theta$, when $\nu_\theta$ is evaluated by formula (12). Putting $e_{30} = e$, it can be shown that

$$n'_\theta - n_\theta = \log_{10}(1 + e - e_\theta) \tag{17}$$

where $e_\theta$ satisfies to the first order the relation derived from equation (10A) and the definition of $n_\theta$

$$\log_{10} e_\theta = \log_{10} e - 2n_\theta \tag{18}$$

Thus to evaluate $n'_\theta$ one needs to consider only the quantities $e$ and $n_\theta$. The values of $e$ are given for each liquid in tabular form in Figures 6 to 17, inclusive.
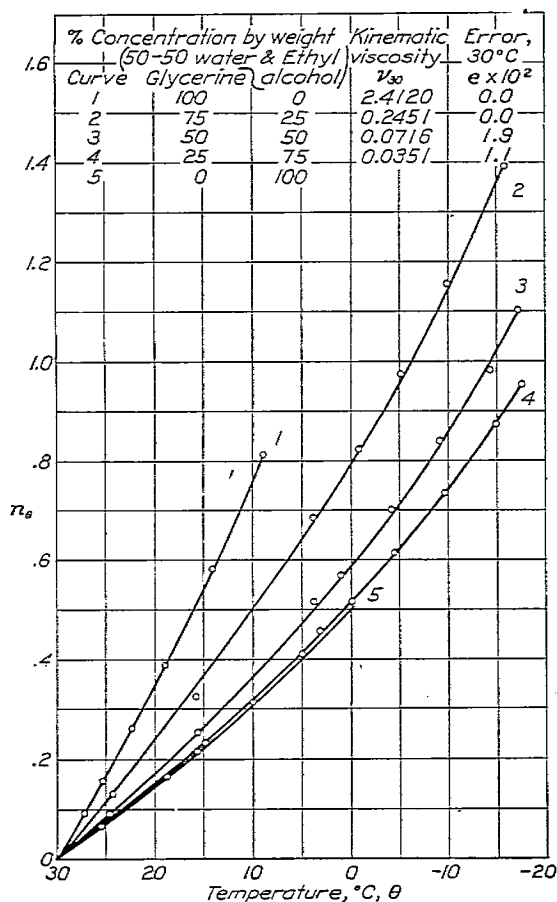


| Curve | % Concentration by weight (50-50 water & Ethyl Glycerine / alcohol) | | Kinematic viscosity $\nu_{30}$ | Error, 30°C $e \times 10^2$ |
|---|---|---|---|---|
| 1 | 100 | 0 | 2.4120 | 0.0 |
| 2 | 75 | 25 | 0.2451 | 0.0 |
| 3 | 50 | 50 | 0.0716 | 1.9 |
| 4 | 25 | 75 | 0.0351 | 1.1 |
| 5 | 0 | 100 | | |

FIG. 14.—Change of kinematic viscosity with temperature. Glycerine and 50-50 water and ethyl alcohol

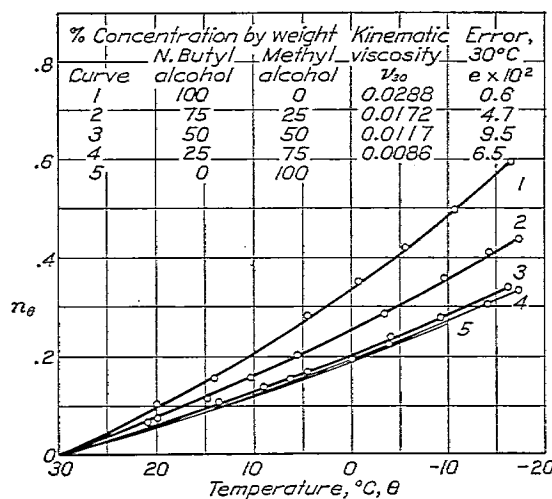| Curve | % Concentration by weight N.Butyl / Methyl alcohol / alcohol | | Kinematic viscosity $\nu_{30}$ | Error, 30°C $e \times 10^2$ |
|---|---|---|---|---|
| 1 | 100 | 0 | 0.0288 | 0.6 |
| 2 | 75 | 25 | 0.0172 | 4.7 |
| 3 | 50 | 50 | 0.0117 | 9.5 |
| 4 | 25 | 75 | 0.0086 | 6.5 |
| 5 | 0 | 100 | | |

FIG. 15.—Change of kinematic viscosity with temperature. N. butyl alcohol and methyl alcohol

As an illustration, suppose that $n'_\theta$, $\nu_\theta$, and $\mu_\theta$ are required for xylene at $\theta = -15°$ C. From the graph for xylene, No. 3, Figure 16, it is seen that $e = 0.038$ and $n_\theta = 0.24$; then by formula (18) $e_\theta = 0.013$, by formula (17) $n'_\theta - n_\theta = 0.01$, and $n'_\theta = 0.25$. The kinematic viscosity $\nu_{-15}$ is given by

$$\nu_{-15} = \nu_{30} \text{ antilog } n'_\theta$$

From the table in Figure 16, $\nu_{30}$ equals 0.0066 and $\nu_{-15} = 0.0117$. From Table VIII $\rho_{30} = 0.855$ and $\alpha = 0.00095$. Then $\rho_{-15} = 0.892$ and $\mu_{-15} = 0.0104$ poise.

Two tables are presented which summarize the experimental work. Table VIII gives data on the viscometers used coupled with the time of flow at 30° and 25° C. for each individual solution The data required for determining the density $\rho_\theta$, namely, $\rho_{30}$ and $\alpha$, are included in this table

Table IX gives the viscosity $\mu$ at 25° C. and the values of the ratio $n$ at $+20°$ and $-15°$ C for each solution. This affords a convenient rough comparison of the values of the viscosity $\mu$ of the liquids at one temperature and of the change of their kinematic viscosity ratio $n$ in the temperature interval $-15°$ to $+20°$ C. The solution corresponding to any desired value of $\mu$ may be found by interpolation and then $n$ at $-15°$ and $+20°$ C. also found by interpolation thus enabling the proper liquid to be selected. The selection should be made bearing in mind the other properties such as given in the description of the samples.

Comparative data on the various solutions is given in Table X. These data are derived by interpolation from Table IX and Figures 6 to 17, inclusive, and give the composition of the solutions against the viscosity at $+25°$ C., varying by convenient steps. The logarithms of the kinematic viscosity ratio are given for five temperatures, neglecting the error $e$, and also the
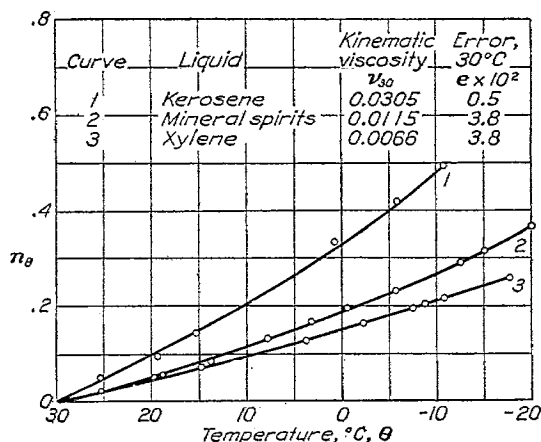


Fig. 16.—Change of kinematic viscosity with temperature. Kerosene, mineral spirits, xylene
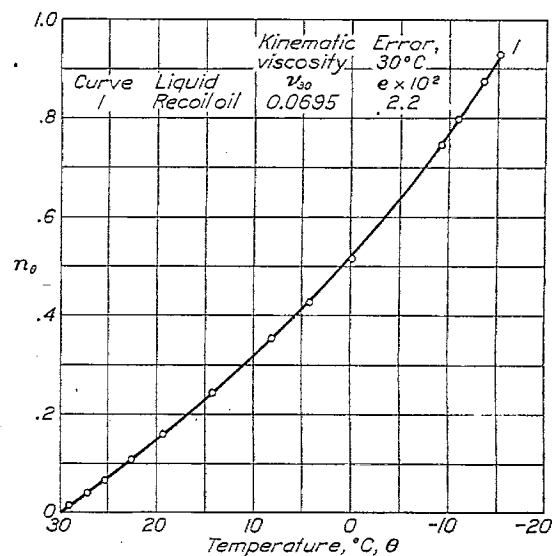


Fig. 17.—Change of kinematic viscosity with temperature. Recoil oil

temperature at which the logarithms of the kinematic viscosity ratio $n$ equals 0.7. At this temperature the viscosity is very closely five times the value at 30° C., which for many purposes sets the lowest temperature at which the liquid can be used.

## SUMMARY

Table X summarizes the data in convenient form. The solutions and liquids of a given viscosity are compared directly as to lowest temperature at which it is useful and the effect of temperature on the kinematic viscosity.

Inclinometers of the rolling-ball type contain for damping a soultion of 40 per cent glycerine and 60 per cent absolute (ethyl) alcohol This solution is seen to have a viscosity between 0.05 and 0.10 (0.065) poise and to become five times as viscous at about $-10°$ C. as at 30° C. A 50-50 solution of N. butyl and methyl alcohol has been proposed for use in order to secure satisfactory operation at lower temperatures. The data in Table X shows that at $-20°$ C. the solution is less than two and one-half times as viscous as at 30° C., but that the viscosity at $+25°$ C. is only one-sixth that of the glycerine-alcohol solution.

Mineral spirits are used to fill magnetic compasses. Only xylene appears to have better temperature properties.

The uses considered above relate to cases in which the liquid is entirely inclosed. When a liquid is desired for use in an open dashpot, other properties, such as evaporation and oxidation, become as important as the effect of temperature. No oily liquid has been found which is by itself satisfactory at low temperatures. It is seen in Table X that all liquids at 25° C. with a

viscosity greater than 0.05 have an excessive temperature effect, the viscosity at −20° C. exceeding more than fivefold that at 30° C. Of the liquids investigated the "wintered" neat's-foot-oil solutions appear to be the best. Their temperature properties are substantially as good as those of other solutions of the same viscosity but these solutions have the advantage of being relatively nonoxidizing. The use of xylene as a solvent introduces the factor of evaporation, but possible solvents with the necessary properties of low viscosity and small change with temperature appear to have an appreciable vapor pressure.

TABLE VIII

DATA FOR EVALUATING $\mu_{25}$ THE VISCOSITY AT 25° C., THE KINEMATIC VISCOSITY AT 30° C., AND $\rho_\theta$ THE DENSITY AT $\theta$° C.

| Proportions, by weight | | Viscometer | | Time of flow | | Density | Thermal coefficient of cubical expansion $\alpha \times 10^3$ |
|---|---|---|---|---|---|---|---|
| | | No. | Mark | $t_{30}$ (sec.) | $t_{25}$ (sec.) | $\rho_{30}$ (gr./cm.³) | |
| Poppy-seed oil: | Xylene: | | | | | | |
| 100 | 0 | $V_3$ | $m_5$ | 64.7 | 79.1 | 0.914 | 0.70 |
| 75 | 25 | $V_3$ | $m_5$ | 15.6 | 17.7 | .900 | .76 |
| 50 | 50 | $V_2$ | $m_5$ | 36.7 | 41.6 | .886 | .82 |
| 25 | 75 | $V_1$ | $m_5$ | 39.2 | 41.6 | .871 | .89 |
| Neat's-foot oil (20° sample): | Xylene: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 36.2 | 44.5 | .905 | .66 |
| 75 | 25 | $V_3$ | $m_5$ | 16.9 | 19.5 | .893 | .73 |
| 50 | 50 | $V_2$ | $m_5$ | 37.5 | 41.6 | .881 | .80 |
| 25 | 75 | $V_1$ | $m_5$ | 40.8 | 43.5 | .869 | .87 |
| Neat's-foot oil (30° sample): | Xylene: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 40.2 | 50.1 | .909 | .70 |
| 75 | 25 | $V_3$ | $m_5$ | 18.1 | 21.4 | .896 | .76 |
| 50 | 50 | $V_2$ | $m_5$ | 37.8 | 41.9 | .883 | .82 |
| 25 | 75 | $V_1$ | $m_5$ | 40.0 | 42.9 | .870 | .89 |
| Castor oil: | Xylene: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 31.6 | 44.7 | .954 | .74 |
| 75 | 25 | $V_3$ | $m_5$ | 62.4 | 79.5 | .928 | .79 |
| 50 | 50 | $V_2$ | $m_5$ | 72.6 | 87.7 | .903 | .85 |
| 25 | 75 | $V_1$ | $m_5$ | 47.3 | 52.7 | .879 | .89 |
| Linseed oil: | Xylene: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 22.8 | 27.0 | .922 | .73 |
| 75 | 25 | $V_3$ | $m_5$ | 13.4 | 15.1 | .905 | .78 |
| 50 | 50 | $V_2$ | $m_5$ | 33.0 | 34.8 | .889 | .84 |
| 25 | 75 | $V_1$ | $m_5$ | 37.1 | 39.5 | .872 | .90 |
| Petrolatum: | Xylene: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 56.1 | 78.0 | .878 | .69 |
| 75 | 25 | $V_3$ | $m_5$ | 11.2 | 13.3 | .871 | .75 |
| 50 | 50 | $V_2$ | $m_5$ | 23.3 | 25.7 | .866 | .82 |
| 25 | 75 | $V_1$ | $m_5$ | 28.9 | 30.8 | .860 | .88 |
| Transformer oil: | Xylene: | | | | | | |
| 100 | 0 | $V_3$ | $m_5$ | 15.0 | 18.4 | .896 | .73 |
| 80 | 20 | $V_2$ | $m_5$ | 36.0 | 40.6 | .887 | .77 |
| 60 | 40 | $V_1$ | $m_5$ | 48.1 | 52.5 | .879 | .82 |
| Glycerine: | Ethyl alcohol: | | | | | | |
| 100 | 0 | $V_4$ | $m_5$ | 148.6 | 217.3 | 1.2455 | .45 |
| 75 | 25 | $V_4$ | $m_5$ | 22.3 | 29.7 | 1.102 | .62 |
| 50 | 50 | $V_3$ | $m_5$ | 12.9 | 15.5 | .993 | .75 |
| 25 | 75 | $V_2$ | $m_5$ | 29.4 | 33.4 | .888 | .88 |
| Glycerine: | Ethyl alcohol and water (50–50): | | | | | | |
| 75 | 25 | $V_4$ | $m_5$ | 15.1 | 19.6 | 1.141 | .55 |
| 50 | 50 | $V_3$ | $m_5$ | 9.7 | 11.8 | 1.052 | .61 |
| 25 | 75 | $V_2$ | $m_5$ | 32.5 | 38.5 | .975 | .68 |
| N. butyl alcohol: | Methyl alcohol: | | | | | | |
| 100 | 0 | $V_1$ | $m_5$ | 70.1 | 78.0 | .811 | .82 |
| 75 | 25 | $V_2$ | $m_5$ | 18.5 | 18.0 | .803 | .90 |
| 50 | 50 | $V_2$ | $m_5$ | 11.8 | 12.6 | .797 | .97 |
| 25 | 75 | $V_1$ | $m_5$ | 22.1 | 23.6 | .789 | 1.05 |
| Xylene | | $V_1$ | $m_5$ | 49.1 | 52.1 | .855 | .95 |
| Kerosene | | $V_1$ | $m_5$ | 74.0 | 82.8 | .810 | .80 |
| Mineral spirits | | $V_1$ | $m_5$ | 28.9 | 30.8 | .769 | .99 |
| Recoil oil | | $V_3$ | $m_5$ | 9.4 | 11.1 | .883 | .74 |

TABLE IX

DATA FOR COMPARING THE VISCOSITY AND THE EFFECT OF TEMPERATURE ON THE VISCOSITY OF THE VARIOUS SOLUTIONS

| Proportions, by weight | | Viscosity $\mu_{25}$ (poise) | Logarithm of the kinematic viscosity ratio | |
|---|---|---|---|---|
| | | | $n_{20}$ | $n_{.15}$ |
| Poppy-seed oil: | Xylene: | | | |
| 100 | 0 | 0. 5460 | 0. 175 | >1. 20 |
| 75 | 25 | . 1197 | . 130 | . 735 |
| 50 | 50 | . 0401 | . 090 | . 500 |
| 25 | 75 | . 0148 | . 055 | •. 335 |
| Neat's-foot oil (20° sample): | Xylene: | | | |
| 100 | 0 | . 6560 | . 180 | >1. 10 |
| 75 | 25 | . 1310 | . 125 | . 770 |
| 50 | 50 | . 0399 | . 085 | . 520 |
| 25 | 75 | . 0156 | . 055 | . 330 |
| Neat's-foot oil (30° sample): | Xylene: | | | |
| 100 | 0 | . 7420 | . 190 | >1. 15 |
| 75 | 25 | . 1433 | . 140 | . 820 |
| 50 | 50 | . 0403 | . 095 | . 525 |
| 25 | 75 | . 0153 | . 065 | . 350 |
| Castor oil: | Xylene: | | | |
| 100 | 0 | 6. 94 | . 335 | >1. 50 |
| 75 | 25 | . 5550 | . 225 | >1. 20 |
| 50 | 50 | . 0867 | . 165 | . 85 |
| 25 | 75 | . 0191 | . 090 | . 600 |
| Linseed oil: | Xylene: | | | |
| 100 | 0 | . 4060 | . 150 | >1. 25 |
| 75 | 25 | . 1023 | . 100 | . 675 |
| 50 | 50 | . 0335 | . 070 | . 475 |
| 25 | 75 | . 0140 | . 055 | . 350 |
| Petrolatum: | Xylene: | | | |
| 100 | 0 | 1. 1150 | . 290 | >1. 25 |
| 75 | 25 | . 0866 | . 150 | . 880 |
| 50 | 50 | . 0239 | . 085 | . 495 |
| 25 | 75 | . 0106 | . 055 | . 325 |
| Transformer oil: | Xylene: | | | |
| 100 | 0 | . 1238 | . 188 | 1. 16 |
| 80 | 20 | . 0392 | . 110 | . 695 |
| 60 | 40 | . 0190 | . 075 | . 475 |
| Glycerine: | Ethyl alcohol: | | | |
| 100 | 0 | 4. 400 | . 345 | --------- |
| 75 | 25 | . 5325 | . 255 | 1. 44 |
| 50 | 50 | . 1151 | . 165 | . 985 |
| 25 | 75 | . 0321 | . 110 | . 650 |
| Glycerine: | Ethyl alcohol and water (50–50): | | | |
| 75 | 25 | . 3640 | . 240 | 1. 36 |
| 50 | 50 | . 0924 | . 175 | 1. 02 |
| 25 | 75 | . 0408 | . 155 | . 885 |
| N. butyl alcohol: | Methyl alcohol: | | | |
| 100 | 0 | . 0263 | . 095 | . 565 |
| 75 | 25 | . 0152 | . 085 | . 410 |
| 50 | 50 | . 0101 | . 060 | . 330 |
| 25 | 75 | . 00733 | . 055 | . 310 |
| Xylene_____ | _____ | . 00595 | . 047 | . 240 |
| Kerosene_____ | _____ | . 0278 | . 095 | >. 500 |
| Mineral spirits_____ | _____ | . 00940 | . 055 | . 315 |
| Recoil oil_____ | _____ | . 0728 | . 146 | . 918 |

TABLE X

| Absolute viscosity 25° C. | Liquid | Temperature at which $n=0.7$ | Logarithm of kinematic viscosity ratio | | | | |
|---|---|---|---|---|---|---|---|
| | | | $n_{30}$ | $n_{10}$ | $n_0$ | $n_{-10}$ | $n_{-20}$ |
| Poises: 6. 0 | 9S per cent D | ° C. 9. 5 | 0 | 0. 6S | | | |
| 4. 0 | 96 per cent D | 8. 5 | 0 | . 66 | | | |
| | 9S per cent H | 11. 0 | 0 | | | | |
| | 99 per cent K | 11. 0 | 0 | . 74 | | | |
| 2. 0 | 90 per cent D | 7. 0 | 0 | . 60 | | | |
| | 90 per cent H | 8. 5 | 0 | . 65 | | | |
| | 96 per cent K | 9. 5 | 0 | . 69 | | | |
| 1. 0 | 83 per cent D | 5. 5 | 0 | . 53 | 0. 89 | | |
| | 99 per cent F | 7. 5 | 0 | . 59 | . 96 | | |
| | 82 per cent H | 6. 0 | 0 | . 58 | . 91 | | |
| | 90 per cent K | 8. 0 | 0 | . 63 | | | |
| . 742 | 30° neat's-foot oil | 0 | 0 | . 42 | . 70 | 1. 1 | |
| . 656 | 20° neat's-foot oil | −2. 0 | 0 | . 39 | . 64 | . 96 | |
| . 6 | 99 per cent B | −2. 5 | 0 | . 39 | . 63 | . 95 | |
| | 97 per cent C | −1. 0 | 0 | . 40 | . 68 | 1. 05 | |
| | 76 per cent D | +3. 0 | 0 | . 48 | . 80 | | |
| | 77 per cent H | 5. 0 | 0 | . 55 | . 87 | | |
| | 82 per cent K | 5. 5 | 0 | . 55 | . 87 | | |
| | 94 per cent F | 4. 0 | 0 | . 50 | . 81 | | |
| . 546 | Poppy-seed oil | −4. 0 | 0 | . 36 | . 59 | . 92 | |
| . 4 | 94 per cent A | −5. 8 | 0 | . 34 | . 56 | . 84 | |
| | 95 per cent B | −4. 0 | 0 | . 37 | . 60 | . 89 | |
| | 91 per cent C | −3. 0 | 0 | . 38 | . 62 | . 94 | |
| | 71 per cent D | +2. 0 | 0 | . 46 | . 76 | | |
| | 90 per cent F | 1. 0 | 0 | . 48 | . 80 | | |
| | 70 per cent H | 2. 5 | 0 | . 49 | . 77 | | |
| | 75 per cent K | 3. 0 | 0 | . 50 | . 79 | 1. 14 | |
| . 4 | Linseed oil | −6. 0 | 0 | . 32 | . 53 | . 85 | |
| . 2 | 82 per cent A | −9. 8 | 0 | . 28 | . 48 | . 70 | |
| | 83 per cent B | −8. 5 | 0 | . 31 | . 49 | . 74 | |
| | 81 per cent C | −7. 2 | 0 | . 33 | . 53 | . 78 | |
| | 63 per cent D | −1. 0 | 0 | . 41 | . 69 | 1. 05 | |
| | 87 per cent E | −10. 0 | 0 | . 27 | . 45 | . 70 | |
| | 85 per cent F | −2. 5 | 0 | . 40 | . 65 | . 94 | |
| | 60 per cent H | −1. 0 | 0 | . 41 | . 67 | . 96 | |
| | 62 per cent K | −1. 0 | 0 | . 43 | . 68 | . 99 | |
| . 1238 | Transformer oil | −3. 0 | 0 | . 40 | . 65 | . 96 | 1. 5 |
| . 1 | 72 per cent A | −14. 5 | 0 | . 25 | . 41 | . 60 | |
| | 70 per cent B | −15. 0 | 0 | . 25 | . 42 | . 61 | |
| | 65 per cent C | −15. 0 | 0 | . 26 | . 42 | . 59 | . 82 |
| | 50 per cent D | −5. 0 | 0 | . 34 | . 56 | . 90 | |
| | 75 per cent E | −16. 0 | 0 | . 24 | . 38 | . 57 | |
| | 78 per cent F | −6. 5 | 0 | . 34 | . 54 | . 79 | |
| | 98 per cent G | −2. 5 | 0 | . 39 | . 63 | . 93 | |
| | 48 per cent H | −6. 0 | 0 | . 34 | . 56 | . 81 | |
| | 53 per cent K | −3. 5 | 0 | . 38 | . 61 | . 89 | |
| . 0728 | Recoil oil | −7. 5 | 0 | . 32 | . 52 | . 76 | |
| . 05 | 57 per cent A | [1]−20. 0 | 0 | . 20 | . 34 | . 48 | . 66 |
| | 57 per cent B | [1]−20. 0 | 0 | . 21 | . 34 | . 50 | . 69 |
| | 52 per cent C | [1]−20. 0 | 0 | . 20 | . 32 | . 47 | . 64 |
| | 44 per cent D | − 8. 5 | 0 | . 31 | . 51 | . 75 | |
| | 60 per cent E | [1]−20. 0 | 0 | . 20 | . 32 | . 46 | . 64 |
| | 63 per cent F | −17. 0 | 0 | . 24 | . 39 | . 55 | . 78 |
| | 86 per cent G | 11. 0 | 0 | . 28 | . 45 | . 67 | |
| | 35 per cent H | −12. 5 | 0 | . 28 | . 46 | . 65 | |
| | 30 per cent K | 7. 5 | 0 | . 33 | . 53 | . 77 | |
| . 0278 | Kerosene | −15. 0 | 0 | . 20 | . 33 | . 48 | |

TABLE X—Continued

| Absolute viscosity 25° C. | Liquid | Temperature at which $n=0.7$ | Logarithm of kinematic viscosity ratio | | | | |
|---|---|---|---|---|---|---|---|
| | | | $n_{30}$ | $n_{10}$ | $n_0$ | $n_{-10}$ | $n_{-20}$ |
| Poises—Contd. | | ° C. | | | | | |
| 0. 025 | 38 per cent A | [1]—20. 0 | 0 | 0. 16 | 0. 25 | 0. 36 | 0. 49 |
| | 35 per cent B | [1]—20. 0 | 0 | . 15 | . 24 | . 35 | . 49 |
| | 38 per cent C | [1]—20. 0 | 0 | . 17 | . 27 | . 38 | . 52 |
| | 30 per cent D | —16. 0 | 0 | . 22 | . 37 | . 55 | 0 |
| | 42 per cent E | [1]—20. 0 | 0 | . 15 | . 25 | . 37 | . 51 |
| | 52 per cent F | [1]—20. 0 | 0 | . 19 | . 31 | . 44 | . 61 |
| | 68 per cent G | [1]—20. 0 | 0 | . 19 | . 32 | . 47 | |
| | 18 per cent H | [1]—20. 0 | 0 | . 20 | . 32 | . 47 | . 63 |
| | 15 per cent K | —10. 0 | 0 | . 31 | . 50 | . 70 | |
| | 98 per cent L | —21. 5 | 0 | . 20 | . 32 | . 46 | . 66 |
| . 01 | 15 per cent C | [1]—20. 0 | 0 | . 12 | . 19 | . 26 | . 41 |
| | 10 per cent D | [1]—20. 0 | 0 | . 13 | . 22 | . 32 | . 44 |
| | 20 per cent G | [1]—20. 0 | 0 | . 12 | . 19 | . 26 | . 35 |
| | 50 per cent L | [1]—20. 0 | 0 | . 13 | . 20 | . 28 | . 37 |
| . 01 | Ethyl alcohol | [1]—20. 0 | 0 | . 16 | . 24 | . 33 | . 43 |
| . 094 | Mineral spirits | [1]—20. 0 | 0 | . 12 | . 19 | . 27 | . 37 |
| . 00595 | Xylene | [1]—20. 0 | 0 | . 10 | . 15 | . 21 | . 26 |

[1] Below.

A____ Poppy-seed oil and xylene.
B____ Neat's-foot oil (20°F.) and xylene.
C____ Neat's-foot oil (30°F.) and xylene.
D____ Castor oil and xylene.
E____ Linseed oil and xylene.
F____ Petrolatum and xylene.

G____ Transformer oil and xylene.
H____ Glycerine and ethyl alcohol.
K____ Glycerine and 50–50 solution of ethyl alcohol and water.
L____ N. butyl alcohol and methyl alcohol.

The percentage given in the table refers to the amount, by weight, of the first-named liquid.

BUREAU OF STANDARDS,
        WASHINGTON, D. C., *May 16, 1928.*

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